11/19/19

Outline for rest of semester

11/20 Test 12

11/21 Class (finish M1-S10)

11/26 No Class

12/3 Test 13 Review

12/4 Test 13

12/5 Return Test 13

MISIO (Continued)

"Permanent" Disability Model

$$\begin{tikzcd}
(0) \arrow[r] \arrow[dr] & (1) \arrow[d] \\
& (2)
\end{tikzcd}$$

Trivial Probabilities include

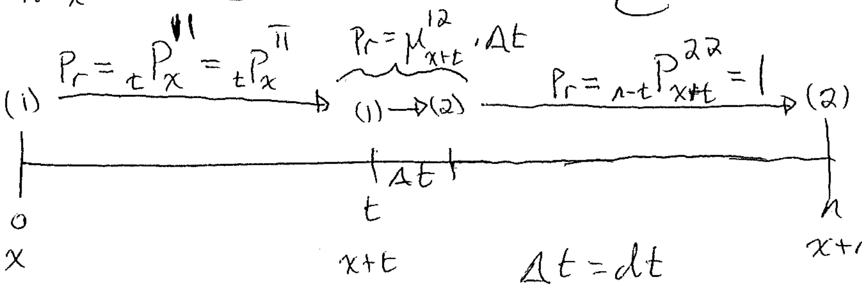
$${}_nP_x^{22} = 1, \quad {}_nP_x^{10} = 0, \quad \ldots$$

Nontrivial Probabilities

$${}_nP_x^{11} = {}_nP_x^{\overline{11}} = e^{-\int_0^n \mu_{x+t}^{12}\,dt} = e^{-\int_0^n \mu_{x+t}^{12}\,dt}$$

$${}_nP_x^{00} = {}_nP_x^{\overline{00}} = e^{-\int_0^n \mu_{x+t}^{0\bullet}\,dt} = e^{-\int_0^n (\mu_{x+t}^{01} + \mu_{x+t}^{02})\,dt}$$

${}_nP_x^{12}$:



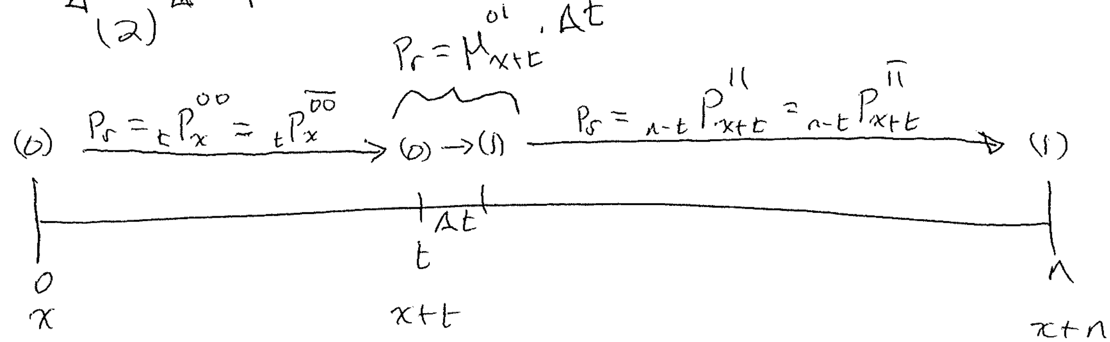
$$\therefore \; {}_nP_x^{12} = \int_0^n {}_tP_x^{\overline{11}} \cdot \mu_{x+t}^{12}\,dt$$

e.g. if $\mu_x^{12} = 0.04$, then this becomes

$${}_{10}P_x^{12} = \int_0^{10} e^{-.04t} \cdot (.04)\,dt = e^{-.04t}\Big|_{10}^{0} = 1 - e^{-.4}$$

$$\begin{tikzcd}
(0) \arrow[rr, "\mu^{01}=.1"] \arrow[dr, "\mu^{02}=.2"'] & & (1) \arrow[dl, "\mu^{12}=.4"] \\
& (2) &
\end{tikzcd}$$

${}_nP_x^{01}$:



$$\therefore\ {}_nP_x^{01} = \int_0^n {}_tP_x^{\overline{00}} \cdot \overset{0.1}{\mu^{01}_{x+t}}\ {}_{n-t}P_{x+t}^{\overline{11}}\ dt$$

$${}_tP_x^{\overline{00}} = e^{-\int_0^t (0.1+0.2)\,dr} = e^{-0.3t}$$

$${}_{n-t}P_{x+t}^{\overline{11}} = e^{-\int_0^{n-t} (0.4)\,dr} = e^{-0.4(n-t)}$$

$$\therefore\ {}_nP_x^{01} = \int_0^n e^{-0.3t} \cdot (0.1) \cdot \overbrace{e^{-0.4n} \cdot e^{0.4t}}\ dt$$

$$= 0.1\, e^{-0.4n} \cdot \int_0^n e^{0.1t}\, dt$$

$$= \cancel{0.1}\, e^{-0.4n} \cdot \frac{1}{\cancel{0.1}}\, e^{0.1t}\Big|_0^n$$

$$= e^{-0.4n} \cdot \left(e^{0.1n} - 1\right)$$

$$= e^{-0.3n} - e^{-0.4n}$$

$$\begin{tikzcd}
(0) \arrow[r] \arrow[dr] & (1) \arrow[d] \\
& (2)
\end{tikzcd}$$

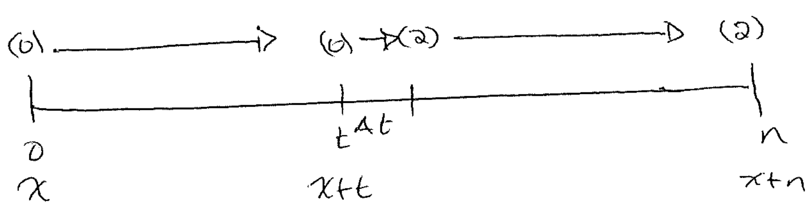
${}_nP_x^{02}$ (easy way (indirect) is use the fact that

$${}_nP_x^{00} + {}_nP_x^{01} + {}_nP_x^{02} = 1 \;)$$

Directly, the picture looks like:

$${}_nP_x^{02} : \quad \underbrace{(0) \longrightarrow (2)}_{I_1 = \Pr \text{ of this}} \qquad \text{or} \qquad \underbrace{(0) \longrightarrow (1) \longrightarrow (2)}_{I_2 = \Pr \text{ of}}$$

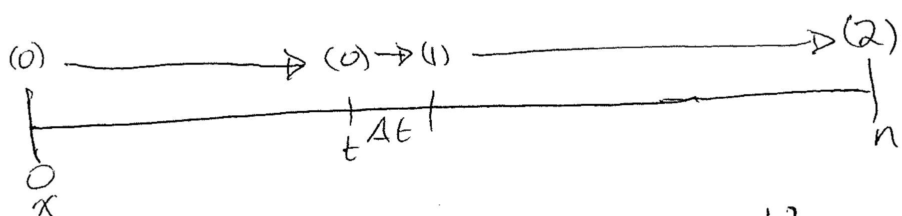
I_1 :



$$\Pr = {}_tP_x^{\overline{00}} \cdot \mu_{x+t}^{02} \cdot \Delta t \cdot 1$$

$$\therefore I_1 = \int_0^n {}_tP_x^{\overline{00}} \cdot \mu_{x+t}^{02} \cdot dt$$

I_2 :



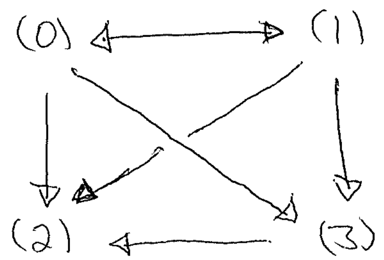
$$\Pr = {}_tP_x^{\overline{00}} \cdot \mu_{x+t}^{01} \cdot \Delta t \cdot {}_{n-t}P_{x+t}^{12}$$

$$\text{where } {}_{n-t}P_{x+t}^{12} = \int_0^{n-t} {}_sP_{x+t}^{11} \cdot \mu_{x+t+s}^{12} \cdot ds$$

$$\therefore I_2 = \int_0^n {}_tP_x^{\overline{00}} \cdot \mu_{x+t}^{01} \cdot \int_0^{n-t} {}_sP_{x+t}^{11} \cdot \mu_{x+t+s}^{12} \, ds \, dt$$

Let's look at more complicated models.

E.g.



Note that many of the ${}_nP_x^{ij}$'s cannot be exactly calculated. E.g.

${}_nP_x^{00}$ is not doable since there are infinitely many paths from (0) to (0); namely

#1) (0) stay is state (0)

#2) $(0) \longrightarrow (1) \longrightarrow (0)$

#3) $(0) \longrightarrow (1) \longrightarrow (0) \longrightarrow (1) \longrightarrow (0)$

$\vdots \qquad \vdots$

In this case we use "Euler's Method" to approximate these probabilities, noting that with $n=0$, we have

$${}_0P_x^{ij} = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{otherwise} \end{cases}$$

Recall

$$y'(t) = \frac{d}{dt}\left[y(t)\right] = \dot{y}(t)$$

Newton's Notation

$$\dot{y}(t) \approx \cancel{\lim_{h \to 0}} \frac{y(t+h) - y(t)}{h}$$

$$\therefore \quad y(t+h) = y(t) + h \cdot \dot{y}(t) \qquad \text{EM}$$